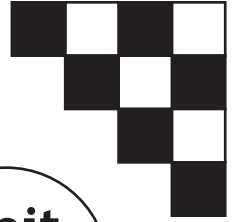


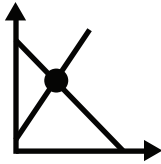
# Mathematics 10C



## Student Workbook

### Unit 6

$$y = x + 1$$
$$y = -x + 4$$



### Lesson 1: Solving Systems Graphically

Approximate Completion Time: 3 Days

$$x = 2y - 2$$
$$3x - y = 4$$

$3(2y - 2) - y = 4$

### Lesson 2: Substitution

Approximate Completion Time: 3 Days

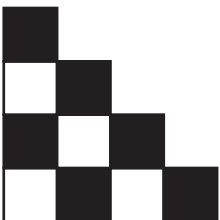
$$2x - y = -6$$
$$-(2x + 4y = -16)$$

---

$$-5y = 10$$

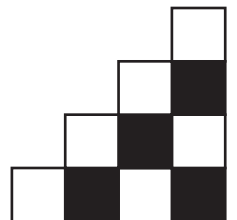
### Lesson 3: Elimination

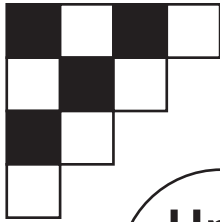
Approximate Completion Time: 3 Days



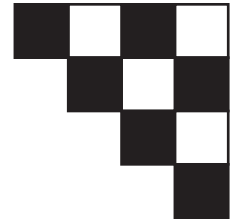
## UNIT SIX

### Systems of Equations





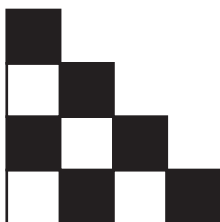
# Mathematics 10C



Unit  
**6**

**Student Workbook**

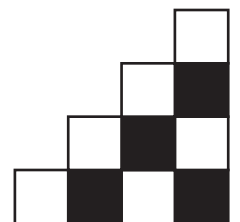
Complete this workbook by watching the videos on [www.math10.ca](http://www.math10.ca).  
Work neatly and use proper mathematical form in your notes.



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## UNIT SIX

### Systems of Equations





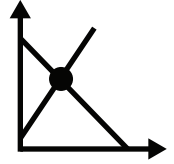
# Systems of Equations

## LESSON ONE - Solving Systems Graphically

### Lesson Notes

$$y = x + 1$$

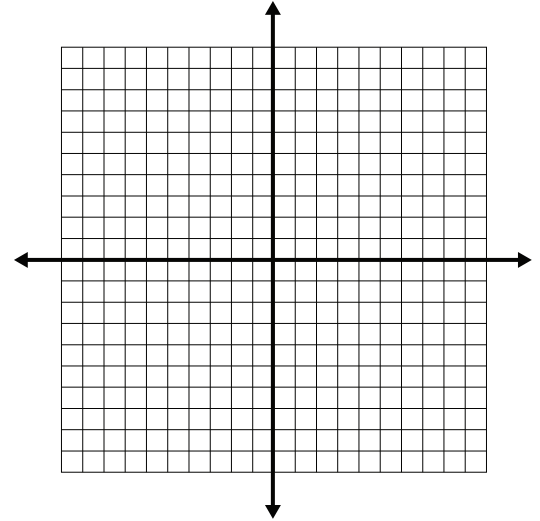
$$y = -x + 4$$



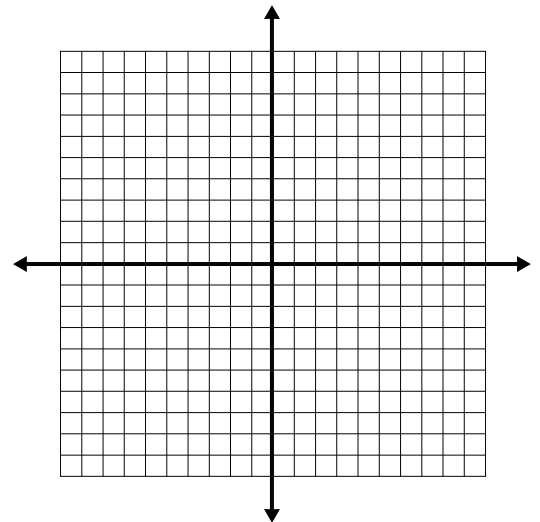
#### Example 1

Solve graphically.

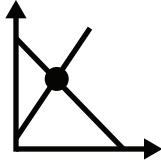
a)  $y = -\frac{3}{4}x + 1$  and  $y = \frac{7}{4}x - 9$



b)  $4x - 7y + 35 = 0$  and  $5x + 7y + 28 = 0$



$$y = x + 1$$
$$y = -x + 4$$



# Systems of Equations

## LESSON ONE - *Solving Systems Graphically*

### Lesson Notes

---

#### Example 2

Determine if each system of equations has one solution, infinite solutions, or no solution.

a)  $y = \frac{1}{2}x + 1$  and  $y = \frac{1}{6}x + 3$

b)  $y = -2x + 3$  and  $6x + 3y = 9$

c)  $y = -\frac{1}{3}x + 6$  and  $2x + 6y = 24$

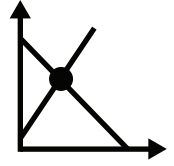
# Systems of Equations

## LESSON ONE - *Solving Systems Graphically*

### Lesson Notes

$$y = x + 1$$

$$y = -x + 4$$



#### Example 3

Determine the number of solutions for each system by inspecting the coefficients.

a)  $x + 2y = 8$  and  $x + 2y = 8$

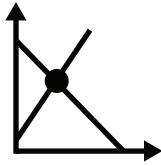
b)  $3x + 9y = -9$  and  $x + 3y = -3$

c)  $x + 2y = 4$  and  $x + 2y = 10$

d)  $4x + 12y = 12$  and  $x + 3y = 9$

$$y = x + 1$$

$$y = -x + 4$$



# Systems of Equations

## LESSON ONE - Solving Systems Graphically

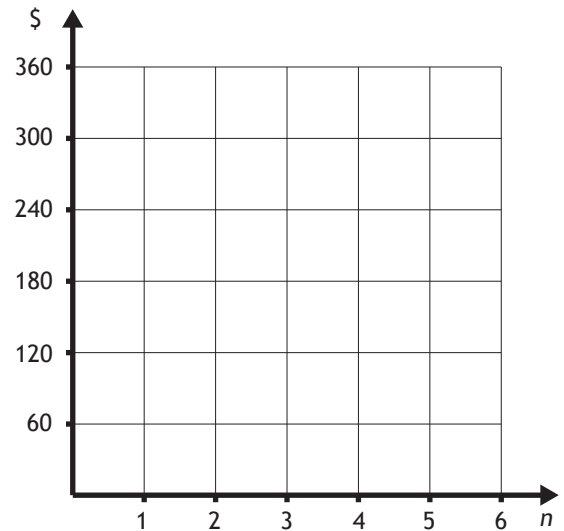
### Lesson Notes

#### Example 4

Four students, Anne, Bethany, Clyde, and Daniel, are raising money in a school fundraiser. Their current total and donation rate are shown in the following table:

	Current Total	Donation Rate
Anne	\$240	\$20/week
Bethany	\$120	\$30/week
Clyde	\$60	\$30/week
Daniel	\$0	\$60/week

a) write equations for each student and graph each line on the same grid.



b) How many weeks will it take for Daniel to catch up to Bethany?

c) Will Daniel ever raise more money than Anne?

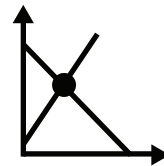
# Systems of Equations

## LESSON ONE - Solving Systems Graphically

### Lesson Notes

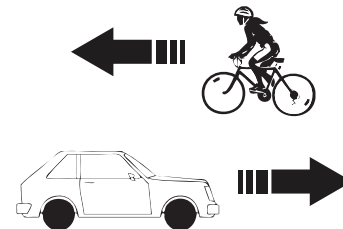
$$y = x + 1$$

$$y = -x + 4$$



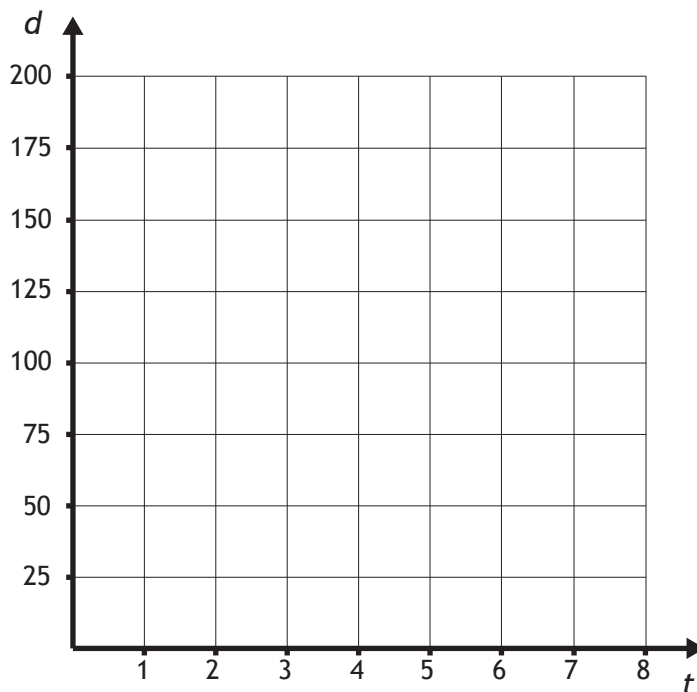
### Example 5

The highway distance from Edmonton to Edson is 200 km. Heidi leaves Edmonton at noon (on her bike) and averages 25 km/h. Cameron leaves Edson (by car) at exactly the same time, and drives at 100 km/h.



Equation one:

Equation two:



a) how long will it take for Heidi and Cameron to pass on the highway?

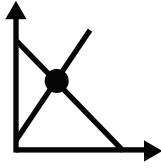
b) how far away from Edmonton are Heidi and Cameron when they pass?

Complete the algebraic solution here



$$y = x + 1$$

$$y = -x + 4$$



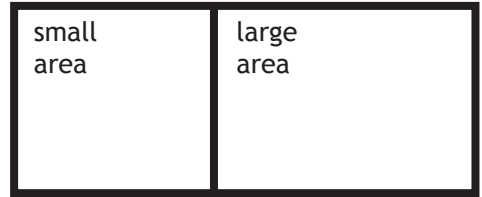
# Systems of Equations

## LESSON ONE - Solving Systems Graphically

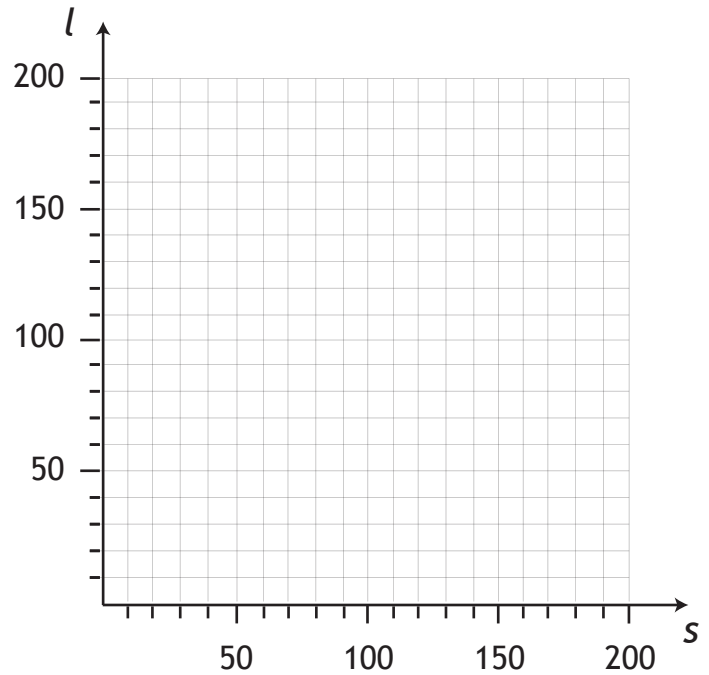
### Lesson Notes

### Example 6

A rectangular lot is separated by a fence. The large region has an area  $20 \text{ m}^2$  greater than the small region. The total area of the lot is  $145 \text{ m}^2$ . Determine the area of each region.



Equation one:	Equation two:



Complete the algebraic solution here

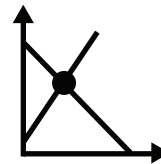
# Systems of Equations

## LESSON ONE - *Solving Systems Graphically*

### Lesson Notes

$$y = x + 1$$

$$y = -x + 4$$

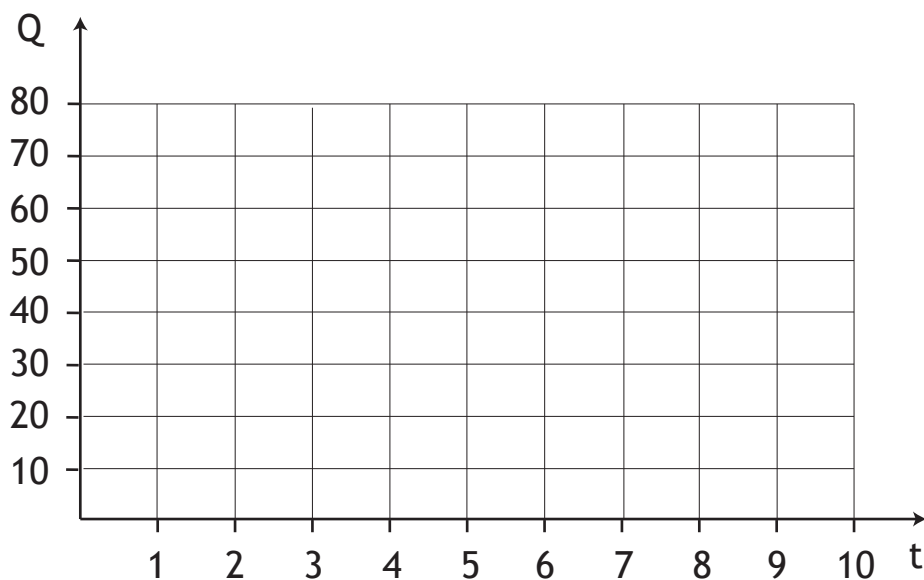


### Example 7

Peter and Nancy are writing a math workbook. Peter has already written 28 questions and can write 4 questions/hour. Nancy has already written 20 questions and can write 5 questions/hour.

Equation one:

Equation two:



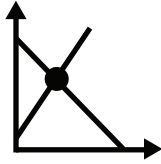
Complete the algebraic solution here

a) when will both writers have written the same number of questions?

b) how many questions will have been written in total?

$$y = x + 1$$

$$y = -x + 4$$



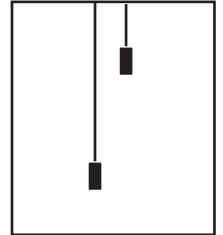
# Systems of Equations

## LESSON ONE - Solving Systems Graphically

### Lesson Notes

### Example 8

In an apartment building, one elevator rises from the 14th floor to the 24th floor in 20 seconds. During that same time, another elevator descends from the 32nd floor to the 12th floor.



Equation one:

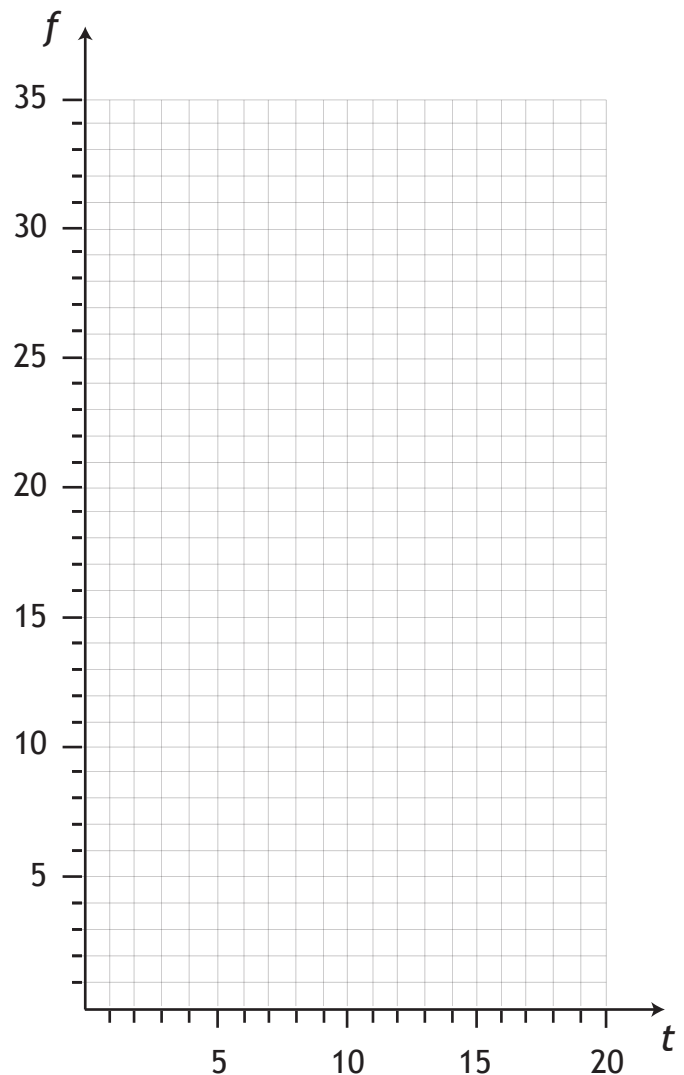
Equation two:

a) graph the motion of each elevator and provide equations.

b) how many seconds will it take for the elevators to pass each other?

c) on what floor will the elevators pass?

Complete the algebraic solution here



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$$x = 2y - 2 \quad 3x - y = 4$$

$$3(2y - 2) - y = 4$$

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

### Introduction

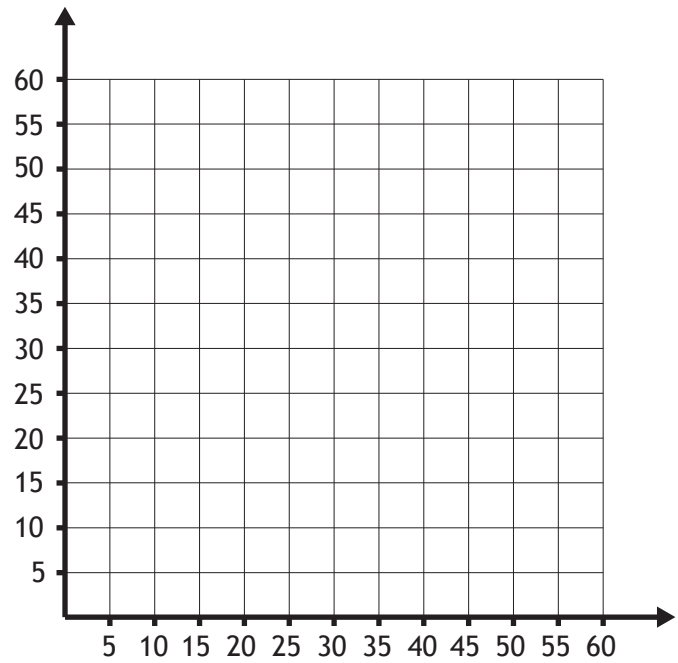
A 60 m cable is cut into two pieces. One piece is twice as long as the other piece. Determine the length of each piece of cable.



Equation one:	Equation two:

a) Solve the system graphically.

b) Solve the system using substitution



# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

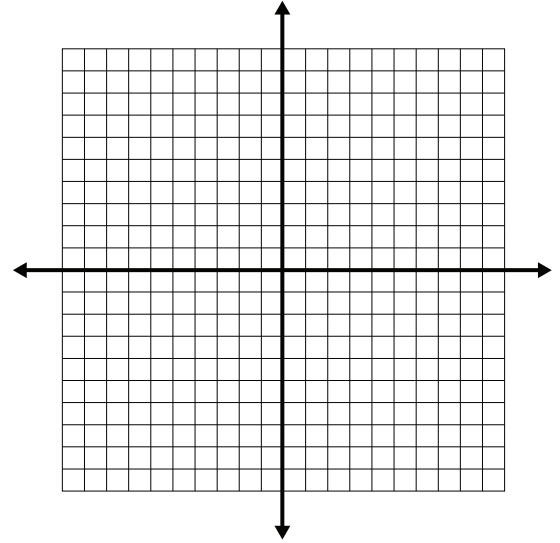
$$x = \underline{2y - 2} \quad 3x - y = 4$$

$3(2y - 2) - y = 4$

#### Example 1

Solve the system  $x + 3y = 9$  and  $4x - y = 10$

a) graphically



b) using substitution

#### Example 2

Solve each of the following systems using substitution.

a)  $x - 2y = -2$  and  $3x - y = 4$

$$x = 2y - 2 \quad 3x - y = 4$$

$$3(2y - 2) - y = 4$$

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

b)  $x + 6y = -29$  and  $x + \frac{1}{4}y = -6$

#### Independent Systems

Systems that yield a definite result, such as  $x = -5$  and  $y = -4$ , are called ***independent systems***.

The equations of an independent system yield intersecting lines and have one solution.

*Memorize this!*

c)  $x + 3y = 3$  and  $3x + 9y = 9$

#### Indeterminate Systems

Systems that yield an equality, such as  $0 = 0$  or  $2 = 2$ , are called ***indeterminate systems***.

The equations of an indeterminate system yield identical lines and have infinite solutions.

*Memorize this!*

d)  $2x - y = -13$  and  $2x - y = -1$

#### Inconsistent Systems

Systems that yield a false result, such as  $0 = 12$ , are called ***inconsistent systems***.

The equations of an inconsistent system yield parallel lines and have no solution.

*Memorize this!*

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

---

$$\begin{array}{l} x = 2y - 2 \qquad 3x - y = 4 \\ \xrightarrow{\hspace{1.5cm}} \\ 3(2y - 2) - y = 4 \end{array}$$

### Example 3

Katrina has \$2.50 worth of nickels and dimes. She has 36 coins in total.  
How many nickels and dimes does she have?

	nickels	dimes	total
number of coins			
monetary value			

Equation one:	Equation two:
---------------	---------------



# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

$$x = 2y - 2 \quad 3x - y = 4$$

$$3(2y - 2) - y = 4$$

#### Example 4



Kory drives from Edmonton to Lloydminster and back. Going to Lloydminster, he drives with an average speed of 96 km/h. For the return trip, he averages a speed of 100 km/h. The total time driving is 5.1 hours. Using this information, calculate the distance from Edmonton to Lloydminster.

	distance	speed	time
going			
returning			

Equation one:

Equation two:

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

$$\begin{array}{l} x = 2y - 2 \qquad 3x - y = 4 \\ \xrightarrow{\hspace{1.5cm}} \\ 3(2y - 2) - y = 4 \end{array}$$

### Example 5

Nathan scored 76% on the multiple choice portion of his physics test and 62% on the written portion. For the entire test, Nathan scored 50 points out of a possible 75. How many marks was each portion of the test worth?

	multiple choice	written	whole test
Possible Points			
Nathan's Points			

Equation one:	Equation two:

$$x = 2y - 2 \quad 3x - y = 4$$

$$3(2y - 2) - y = 4$$

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

### Example 6



James invests a total of \$5000 in two different investments. The first investment earns 2.9% interest, and the second investment earns 4.5% interest. The total interest earned is \$196.20. How much did James invest in each investment?

	lower yield inv.	higher yield inv.	total
money invested			
interest earned			

Equation one:	Equation two:

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

$$\begin{array}{l} x = 2y - 2 \qquad 3x - y = 4 \\ \xrightarrow{\qquad \qquad \qquad} \\ 3(2y - 2) - y = 4 \end{array}$$

### Example 7

One bin of dried fruit mix contains 28% apricots. A different bin of dried fruit mix contains 18% apricots. A new mix is made using one scoop from each bin. This mix has a mass of 600 g, and contains 25% apricots. What was the mass of dried fruit in each scoop?

	bin 1	bin 2	new mix
mass of dried fruit in scoop			
mass of apricots			

Equation one:	Equation two:
---------------	---------------

# Systems of Equations

## LESSON TWO - *Substitution Method*

### Lesson Notes

---

$$x = \underline{2y - 2} \quad 3x - y = 4$$

$$3(2y - 2) - y = 4$$

### Example 8

The system of equations  $x + 2y = 0$  and  $x + 5y = b$  has the solution  $(-2, a)$ . Determine the values of  $a$  and  $b$ .

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$$\begin{array}{r}
 2x - y = -6 \\
 - (2x + 4y = -16) \\
 \hline
 -5y = 10
 \end{array}$$

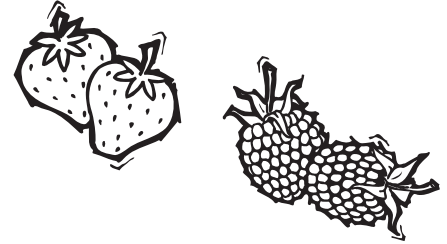
# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

### Introduction

Rahim goes to a grocery store and spends \$22.00 to purchase 3 cartons of strawberries and 2 cartons of raspberries. Paul goes to the same grocery store and spends \$41.00 to purchase 4 cartons of strawberries and 5 cartons of raspberries.



What is the price of one carton of strawberries and one carton of raspberries?

	money spent on strawberries	money spent on raspberries	total spent
Rahim			
Paul			

Equation one:	Equation two:
---------------	---------------

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

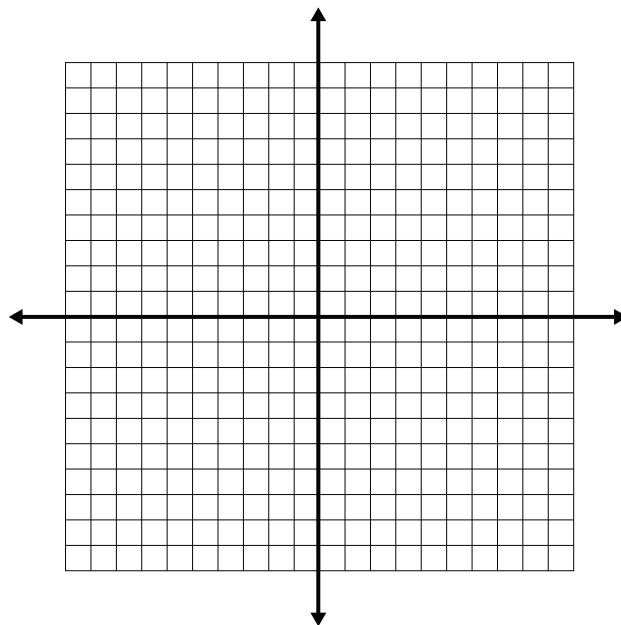
---

$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

#### Example 1

Solve the system  $2x - y = 8$  and  $5x - 3y = 21$

a) graphically



b) using elimination



$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

#### Example 2

Solve each of the following systems using elimination.

a)  $x - \frac{1}{2}y = -3$  and  $\frac{1}{2}x + y = -4$

#### Independent Systems

Systems that yield a definite result, such as  $x = -5$  and  $y = -4$ , are called ***independent systems***.

The equations of an independent system yield intersecting lines and have one solution.

*Memorize this!*

b)  $6x + 4y = 14$  and  $x + \frac{2}{3}y = \frac{7}{3}$

#### Indeterminate Systems

Systems that yield an equality, such as  $0 = 0$  or  $2 = 2$ , are called ***indeterminate systems***.

The equations of an indeterminate system yield identical lines and have infinite solutions.

*Memorize this!*

c)  $x - \frac{1}{2}y = 4$  and  $2x - y = 5$

#### Inconsistent Systems

Systems that yield a false result, such as  $0 = 12$ , are called ***inconsistent systems***.

The equations of an inconsistent system yield parallel lines and have no solution.

*Memorize this!*

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

---

$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

### Example 3

A coin collection has 33 quarters and nickels. The number of nickels is 5 greater than three times the number of quarters. How many coins of each type are there?

Equation one:	Equation two:

a) solve using substitution

b) solve using elimination

$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

#### Example 4



A parking lot contains motorcycles (2 wheels) and cars (4 wheels). There are 35 vehicles and 114 wheels. How many motorcycles and cars are there?

	motorcycles	cars	total
vehicles			
wheels			

Equation one:	Equation two:
---------------	---------------

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

#### Example 5



It takes 3 hours for a canoe to travel 45 km downstream. The return trip, going upstream, takes 5 hours. What is the speed of the boat and the speed of the current?

	distance	speed	time	Equation one:	Equation two:
going (downstream)					
returning (upstream)					

$$\begin{array}{r}
 2x - y = -6 \\
 - (2x + 4y = -16) \\
 \hline
 -5y = 10
 \end{array}$$

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

### Example 6



Tickets to a museum cost \$7 for a child and \$12 for an adult. On a particular day, 233 people attended the museum and there was a total revenue of \$2216. How many tickets of each type were sold?

	child	adult	total	
number of tickets				<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;">Equation one:</div> <div style="width: 45%;">Equation two:</div> </div>   
revenue				

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

---

$$\begin{array}{r} 2x - y = -6 \\ - (2x + 4y = -16) \\ \hline -5y = 10 \end{array}$$

#### Example 7

Corrine's mom is 25 years older than Corrine. In two years, Corrine's mom will be twice Corrine's age. How old are Corrine and Corrine's mom?

$$\begin{array}{r}
 2x - y = -6 \\
 - (2x + 4y = -16) \\
 \hline
 -5y = 10
 \end{array}$$

# Systems of Equations

## LESSON THREE - *Elimination Method*

### Lesson Notes

### Example 8



Ryan and Greg split the driving on a 1335 km trip from Calgary to Winnipeg. Ryan drove to Regina with an average speed of 90 km/h. Greg drove the rest of the way to Winnipeg with an average speed of 100 km/h. The total trip took 14.2 hours.

What is the distance between Calgary and Regina? Regina and Winnipeg?

	distance	speed	time		
Ryan (Calgary to Regina)				Equation one:	Equation two:
Greg (Regina to Winnipeg)					

# Answer Key

## Systems of Equations Lesson One: Solving Systems Graphically

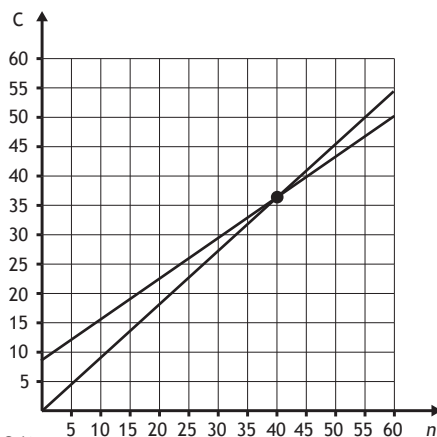
Introduction:

subscription

n	Cost
0	8.00
5	11.50
10	15.00
15	18.50
20	22.00
25	25.50
30	29.00
35	32.50
40	36.00
45	39.50
50	43.00
55	46.50
60	50.00

pay-as-you-go

n	Cost
0	0
5	4.50
10	9.00
15	13.50
20	18.00
25	22.50
30	27.00
35	31.50
40	36.00
45	40.50
50	45.00
55	49.50
60	54.00



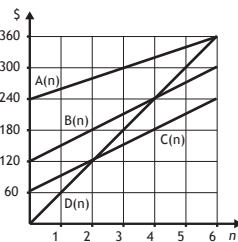
The solution to the system is (40, 36)

The pay-as-you-go option is a better deal for less than 40 downloads.

The subscription option is a better deal for more than 40 downloads.

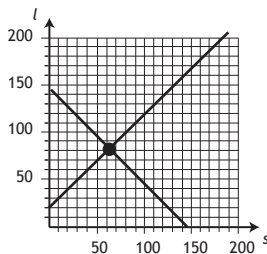
Example 4:

- a)  $A(n) = 20n + 240$ ,  $B(n) = 30n + 120$ ,  
 $C(n) = 30n + 60$ ,  $D(n) = 60n$   
 b) 4 weeks  
 c) yes, in 6 weeks



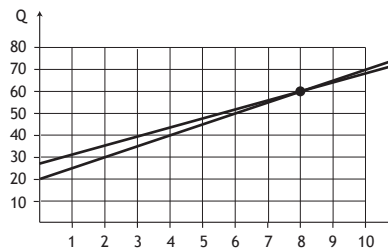
Example 6:

- small area =  $62.5 \text{ m}^2$   
 large area =  $82.5 \text{ m}^2$



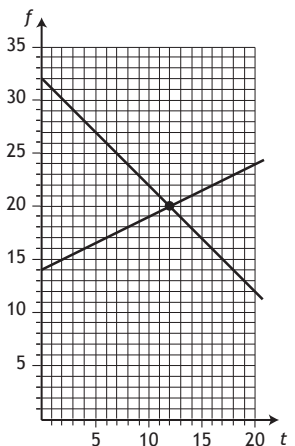
Example 7:

- a) 8 hours  
 b) 120 questions  
 in total from  
 both teachers.



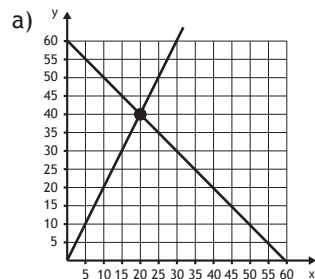
Example 8:

- a) graph below b) 12 s c) 20<sup>th</sup> floor



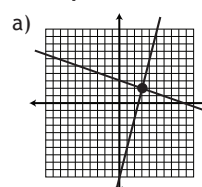
## Systems of Equations Lesson Two: Substitution

Introduction:



- b) the short  
 part of the  
 cable is 20 m,  
 and the long  
 part is 40 m.

Example 1:



- b) solution: (3, 2)

- Example 2: a) (2, 2) b) (-5, -4)  
 c) infinite d) no solution

Example 3: 14 dimes, 22 nickels

Example 4: 250 km

Example 5: Multiple Choice: 25 points, Written: 50 points

Example 6: \$1800 (lower yield), \$3200 (higher yield)

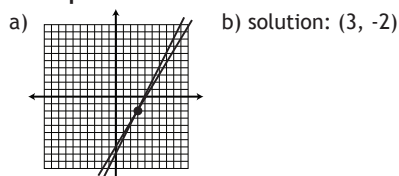
Example 7: scoop 1: 420 g, scoop 2: 180 g

Example 8: a = 1, b = 3

## Systems of Equations Lesson Three: Elimination

Introduction: strawberries: \$4, raspberries: \$5

Example 1:



- Example 2: a) (-4, -2) b) infinite solutions c) no solution

Example 3: 26 nickels, 7 quarters

Example 4: 13 motorcycles, 22 cars

Example 5: canoe: 12 km/h, current: 3 km/h

Example 6: 117 adult tickets, 116 child tickets

Example 7: Corrine: 23, Corrine's mom: 48

Example 8: Calgary to Regina: 765 km, Regina to Winnipeg: 570 km